minus their scalar product. The change suggested would enable students to gradually accustom themselves to the notation of the calculus, which would in fact then form an abridged notation for the cartesian expressions and operations which enter into physical investigations.

I would ask Prof. Knott to give this suggestion his careful consideration, as I am sanguine enough to believe that in it, simple as it appears, lies the possible reconcilement of the new school of vector analysts with the quaternionists. Possibly some symbol other than S would have, at any rate at first, to be employed for this new scalar product. Perhaps, with Prof. Macfarlane, it might be called the cos-product, though that notation properly belongs to the scalar product of two vectors only, and loses its significance if applied to the scalar product of three or more vectors. No single letter symbol could be better than S, as it is distinctive and quick to write. However, the first question is whether there is any possibility of the modification being adopted.

The quaternionic product of a vector by itself would be minus its scalar square, but without any mystery attached to the fact. For the product of two vectors = vector product – scalar product, and therefore, if the vector product is zero, the quaternionic product = – the scalar product. Hence, instead of having

$$(\alpha + \beta)^2 = \alpha^2 + 2\mathrm{S}\alpha\beta + \beta^2,$$

we should have

$$\mathrm{S}(\alpha + \beta)^2 = \mathrm{S}(\alpha^2 + 2\alpha\beta + \beta^2).$$

Reciprocal vectors satisfy the equation $\beta^{-1}\beta = 1$, so that $\mathrm{S}\beta^{-1}\beta = -1$, *i.e.* β^{-1}, β are oppositely directed vectors.

The quaternion

$$\alpha\beta^{-1} = \frac{\alpha\beta}{\beta^2} = \frac{\mathrm{V}\alpha\beta - \mathrm{S}\alpha\beta}{-\mathrm{S}\beta^2} = -\frac{\mathrm{V}\alpha\beta}{\mathrm{S}\beta^2} + \frac{\mathrm{S}\alpha\beta}{\mathrm{S}\beta^2},$$

showing clearly that both the vector and scalar products of $\alpha\beta^{-1}$ are opposite in sign to those of $\alpha\beta$, as must, of course, be the case since β^{-1} and β are oppositely directed vectors. This fact is obscured with the orthodox notation. In fact, so far as I have been able to test the proposed change, I have found no drawbacks, but rather an improvement. ALFRED LODGE.

Sagacity in Horses.

FROM the window opposite, as I write, I have just witnessed an interesting performance on the part of two horses. Bordering the park is a strip of land, doomed to be built upon, but meanwhile lying waste, and used for common pasturage, on which the horses under notice were leisurely grazing. A pony in a cart, having been unwisely left by the owner for a time unattended on the grass, suddenly started off, galloping over the uneven ground at the risk of overturning the cart. The two horses, upon seeing this, immediately joined in pursuit with evident zest. My first supposition, that they were merely joining in the escapade in a frolicsome spirit, was at once disproved by the methodical and business-like manner of their procedure. They soon reached the runaway, by this time on the road, one on one side of the cart, and one the other; then, by regulating their pace, they cleverly contrived to intercept his progress by gradually coming together in advance of him, thus stopping him immediately in the triangular corner they formed. Until the man came up to the pony's head they remained standing thus together quite still; when the two horses, evidently satisfied that all was now right, without any fuss trotted back again together to their grass.

The sagacious conduct of the horses, acting in such perfect cooperation, formed a pretty sight; and it was apparent that, instead of making the pony more excited, they really pacified and calmed him. Why should they not receive "honourable mention" as much as if they were proud human beings?

WILLIAM WHITE.

The Ruskin Museum, Sheffield, June 20.

TERCENTENARY OF THE ADMISSION OF WILLIAM HARVEY TO GONVILLE AND CAIUS COLLEGE, CAMBRIDGE.

BORN at Folkestone, and educated at the King's School, Canterbury, William Harvey was admitted to Gonville and Caius College as a minor scholar in his sixteenth year, on May 31, 1593. The tercentenary of this event was celebrated by Harvey's College on Wednesday, June 21, this being the earliest day after the date of his admission at which rooms were available for those coming from a distance. The guests were received and welcomed by the Master and Fellows, at five o'clock, in the large Combination Room, where tea was provided. In the smaller adjacent room were exhibited a number of objects of interest connected with Harvey, including his pestle and mortar, from the Museum at Folkestone, a rubbing from his mother's tomb, an autograph letter of Harvey, lent by the Master of Sidney Sussex College, and a coloured drawing of Harvey's coat-of-arms, recently discovered on the walls of the buildings of the University of Padua. The latter was presented to the College by the University of Padua, followed on the day of the festivity by a long congratulatory Latin telegram from the Rector, on behalf of the University, which ran as follows:—"Universitatis Patavinæ quæ cum aliis Britannis discipulis tum Harveio Caioque gloriatur, quorum alterius merita insigne Collegium vestrum nunc recolit nomenque ex altero invenit, festi in Harveii honorem indicti participem se profitetur et in renovanda cum celeberrima Universitate Cantabrigiensi vetere studiorum amicitiæque memoria summopere lætatur, pro Academico Senatu, Ferraris Rector." Also an autotype of the panel portrait of Harvey from Rolls Park, Chigwell, Essex, presented to the College by Sir Andrew Clark, as one of a séries of eight, consisting of a central portrait of Harvey's father, surrounded by those of his seven sons. Some early editions of the works of Harvey and of some of his more immediate predecessors and followers were also displayed, together with the admission book of the College, containing the original record of his admission. At seven o'clock the guests assembled once more in the Combination Room, whence they proceeded to dinner in the College Hall, led by the butler, bearing the original "caduceus," as used by Dr. Caius when President of the College of Physicians. The dinner was presided over by the Master of Gonville and Caius College, the Rev. N. M. Ferrers, D.D., F.R.S., above whose chair were displayed a copy of the bust from the Harvey Memorial, crowned with a laurel wreath, and the much-prized portrait of Harvey from the Master's Lodge. After dinner the Grace Anthem of the College, composed by Mr. C. Wood, was sung. The Master then proposed the usual loyal toasts, after which Sir James Paget proposed the toast of the evening, "The Memory of William Harvey."

He remarked that the reason why he had the honour of being asked to propose that toast was his relationship to his brother, who, he believed, made the proposal that there should be that tercentenary of the admission of William Harvey. He desired to remember that, and to speak as he thought his brother would have spoken if he had had the opportunity. He was sure that if he had been present he would have referred to the honour which was due to the college which Caius founded. He would have done that out of the deep sense of gratitude which he had for the College. For it was the Fellowship founded by Caius that led his brother to the study of medicine, and, on the occasion of that Fellowship which he held becoming vacant, to give himself entirely to it. To that he owed a great part of the happiness of his life, and he hoped he (the speaker) would not be deemed wrong if he said that indirectly he himself was also deeply indebted to Caius College, for it was through the large income which was associated with that Fellowship that his brother was enabled, out of his abundant generosity, to help him greatly in the study of his profession at St. Bartholomew's Hospital, of which Harvey was so great an ornament and honour. He wished that they knew more of the time and the work he did in Caius College. Indirectly Harvey owed to Caius himself the opportunity of being a student of the College. It was not, he thought, known whether Harvey was originally destined to be a student of medicine or physic, or whether he was led to it mainly by that which he found in that College, from the help and advantages given to the study of medicine and sciences. Harvey found there—and there alone, he thought, certainly amongst the Colleges of the University—a license for dissection. A license was obtained from the King to dissect in that College the bodies of criminals, and Dr. Venn, in the register of St. Mary's parish, had found records of two who were executed here in Harvey's time. The register said distinctly "They have been buried here after being anatomised in Caius College. He might add that the bodies were to be interred with great reverence, and the Masters and Fellows had to attend the funerals. From that College Harvey went to Padua, where he had the best learning from the best biological teachers of the time. He took the degree of doctor of medicine with the highest honour, and then he returned to the practice of his profession and the teaching of it in the University. Alluding to Harvey's discovery of the circulation of the blood, Sir James said he thought he might venture to say that that was the greatest discovery in biological science ever made by one man. He thought there never had been any one man to whom biology was so indebted as to Harvey for that discovery, and that was in the early part of his life. He supposed they could not now think of what would have been the progress of biology but for that discovery, neither could they form any just estimate of the honour due to Harvey for that discovery, which was to them now so plain, so evident, that one might wonder how it could ever have been doubted, but was then surrounded by difficulties which it seemed impossible ever to overcome. It was marvellous, if one looked back at it, to think what must have been the power of observation, the ingenuity, the constant, resolute industry of the man who could find that out, not only in the face of actual difficulties of inquiry, but in the face of those who were perfectly satisfied with their own opinions. He worked on and on until he brought out the best result he could obtain. He had shown by his discoveries, which had had even a greater influence on the progress of biological knowledge, the right method of inquiry. He had to find his results in the face of that full and perfectly-satisfied belief that all truth in such a science as that of medicine could be deduced from general principles then prevalent, and from the physiological doctrines which few men then dared to doubt. Nothing could have proved more than Harvey's results that the way to knowledge in biological science was through continual observation and experiment and recording. That was what Harvey showed, and it had never been forgotten. Again and again Harvey said in his works, and more especially in that admirable introduction to his work "De Generatione," that the way to knowledge was by observing, experimenting, and recording, and not by thinking. The same thought was expressed by John Hunter, who said, "Don't think; try." Those were words he (Sir James) would venture to say every one pursuing biology might well bear in mind. Both of those men were most earnest and profound thinkers. This could be traced in all their works, but that in which they distinguished themselves from other men of the same calling and the same pursuits was that they tried their thoughts. They tested them by every possible observation and experiment. They thought, and thought, and thought, but they were never satisfied with thinking; every thought they had was tried by experiment. When they remembered that Harvey was not only the greatest physiologist of the time, but the greatest physician, it was well to look and see as far as they could how much he himself followed that out, and he thought it would be found, unhappily, they had scarcely any record of Harvey's observations in practice. Repeatedly in his works that were published he stated that he intended to publish his medical observations. Now the whole of those, he supposed, were lost, and yet his (the speaker's) brother pointed out there was no certain knowledge at all either of the time or of the manner in which they were lost. Those observations would be of inestimable value if they could be found. They might hope that some of the younger Caius men would find out where those manuscripts were. It would be well if the MSS. could be published in *facsimile*, or in the same manner as the one done by the College a few years ago, when they at last found Harvey's lectures on anatomy and surgery. He asked them to drink to the memory of Harvey, who had made discoveries surpassing those ever made by any one man, and had showed the true and only sure methods by which biological science could be increased.

After the toast had been duly honoured the College "Carmen Caianum" was sung, the words of which were written by the President, Rev. B. H. Drury, and set to stirring music by Mr. C. Wood. An extra verse commemorative of Harvey and Glisson, also once a member of the College, was introduced into the song for this occasion.

Dr. Clifford Allbutt (Regius Professor of Physic) proposed the toast of "The Guests," and in doing so read a letter which had been received from the rector of Padua University, in which he expressed the pride of that University at having been the place where William Harvey had pursued his studies. Dr. Allbutt also referred to the sense of loss which was felt by all present at the death of Sir George Paget, who would, had he lived, have been the man of all others upon whom it would have been fitting that the duty of taking an important part in that celebration should have fallen.

Sir Andrew Clark responded, and said he desired in the name of his more distinguished fellow guests to give them his grateful thanks for permitting them to be present that evening.

Prof. Gairdner next proposed "The University," and said that the toast needed nothing on his part to recommend it. He could not conceive a greater eulogium upon the University of Cambridge than that it contained such magnificent representatives of ancient learning as their Vice-Chancellor, Dr. Peile; and at the same time eminent representatives of what he would venture to call the modern scientific method as the Professor of Medicine, Dr. Clifford Allbutt and of Physiology, Prof. Michael Foster. The University was an institution founded upon all that was best in human learning and in human experiments, and it will go on and prosper to the end of time. He proposed the health of the University and coupled with it the name of their distinguished Vice-Chancellor, Dr. Peile.

The Vice-Chancellor, in returning thanks, alluded to the great development of studies and of buildings in the University during the past thirty years. He was sorry to say that their rapid development had almost caused alarm in certain most important quarters. He read the other day one of the leading newspapers of England, which called attention to their unsatisfied spirit of innovation. He thought that that must have reference to some of their most recent developments of the engineering tripos. Yet surely it might have occurred to any one that the sciences of engineering were most closely connected with the study of mathematics, which was their chief glory in Cambridge. Possibly, also, it had reference to the development of agricultural science. Well, agricultural science was a very excellent thing. It seemed to him that, after all, some of the greatest discoveries in science had been made, not merely by students or by lecturers, but by men who had been carrying on professional work and working purely with mercantile aims. The duty of the University, he took it, was to encourage those studies as well as others. But there would always be a problem before them. At present the problem would raise the very, very old story—the limited means of the University. The problem was how far could they encourage new studies, which could never lead those who were following them to any great pecuniary rewards, although they might lead them to the rewards of learning. He thought that was a great problem. This year they had an examination in Oriental languages. There was one candidate for it; they had four examiners, and the cost of that examination, he supposed, was £50 or £60 to the University, which was perfectly right. It was just those studies which could not pay their way, which could never be supported without the help of endowments. He did not think the problem was so serious as it might seem at first. Two years ago, when his predecessor, Dr. Butler, resigned his office, he pointed out some of the needs of the University. Dr. Butler's clear and lucid statement brought forth one magnificent gift and nothing more. He (the speaker) in his turn, in the first year of his office, sent forth "a bitter cry" of the needs of the University. Yet that bitter cry brought forth nothing. Did it seem possible that after all a general cry might not be specially efficacious, while a request for special help might serve for a cry? He was happy to say that this seemed at last to solve the problem of how they could develop the newer studies with the help of those outside who were willing to support them. The engineering school had, by the labours of Prof. Ewing and Mr. Horace Darwin, received money, which he hoped would carry it on sufficiently. He believed so ancient an institution as the Observatory of Cambridge was going to ask for a new telescope to carry on its work. That being so, it seemed that partially at least the problem was solved. The problem concerning agricultural science had been solved by the liberal aid of the County Councils. Those bodies had come to their aid in the most generous manner, and given them enough to carry on their work for at least some years. He hoped, as he said before, they would see their way, not merely to maintain and develop those old institutions which had been from all times the glory of Cambridge, but also to carry on those newer studies and newer developments which would keep them in touch with the nation, and make them remembered for all times, and which, whatever developments might arise elsewhere, would make Cambridge one of our greatest centres of educational life.

In proposing the "Health of the Master and Fellows," the Right Hon. T. H. Huxley, who was enthusiastically received, said he was charged with a very pleasant duty, and one which could be happily performed without either gifts of eloquence or even those of voice, in which unhappily he was at present sadly deficient, and he would not be withdrawn from the simple discharge of that duty by the invitation which had been addressed to him by a previous speaker to enter upon the field of controversy. In proper time and place he imagined that he could hardly be said to have shown any unwillingness for the discussion of controverted questions, but in his judgment they were extremely inappropriate and out of place in a meeting of that kind, and he desired absolutely to abstain from that, and to confine himself to the business in hand, which was of a far more pleasant, and, he ventured to think, more profitable nature: it was to propose to them the health of the Master and Fellows of that College. All those who were present would understand the gratitude which they all felt for the generous and gracious hospitality which they had shown to them on that occasion, but it was a traditional hospitality, and it went back to the time when that important corporation, of which they were the present representatives, extended their hospitality to William Harvey, whose name and fame they were met to celebrate. He did not know whether the Master and Fellows of that time were aware of what they were doing in training and disciplining that young man—boy, indeed, to them—to make the best use of the faculties with which he was endowed, but he thought it lay to their credit that from that time to this, the hospitality which they extended to science—to biological science especially, and to that branch of it which was called the science of medicine very particularly—that that had been continued with unbroken openness and readiness. It was for that reason, he thought, that the large proportion of persons present in that room who were devoted to scientific studies would with the greatest possible cordiality drink the toast which he had to propose. For in this matter Gonville and Caius College occupied a position as isolated as it was honourable. He was aware that the studies of biological sciences, and more especially those which had relation to medicine, could not be cleared of the accusation then made against them of utility to mankind. He admitted to the full the charge that was made against those studies, but the present showed, and the future would show more strongly, that quite apart from the bearing of direct utility, it must be regarded as a happy instinct, if not as a purpose of intelligence, which had led that College for these 300 years to cherish and to promote those studies. It was on that ground they who were so deeply interested in its pursuits felt that they owed a debt of gratitude to the College, and he knew of no reason, except the fact that he once took an active part in those biological matters, which had led to his selection as proposer of the health of the College to them on that occasion. Sir James Paget had fully and exhaustively told them, in that admirable language which he had always at command, the great claims of Harvey upon their respect and veneration. He had justly told them that Harvey regarded himself, not merely as a discoverer, but as a propounder and champion of a new method. Dr. Venn was good enough to tell him before the dinner of a fact of which he (the speaker) was entirely ignorant: that before Harvey's time that College possessed what was called an "anatomer," a gentleman whose duties appeared to have been to dissect bodies, which were given over to him and others, to give the students of the College a practical contact with the nature of things. It was in that respect that modern science differed from ancient science; it was in that respect that Harvey was essentially modern. It was therefore to the wise provision of the founders of that College that they owed the beginning of that movement commenced in this country by Gilbert, followed up in Italy by Galileo, followed up conscientiously here by Harvey himself, which had led to the great modern development of scientific culture. They trusted that the hospitality which had hitherto been extended by that College to purely scientific investigations would be continued upon the lines laid down by Harvey. It might be that Harveys existed among them now, and the only thing they had to hope for, and to wish for was, that those Harveys of the future might not be compelled, as the Harvey of the past, to obtain a higher scientific training by going to the University at Padua. They hoped that in this University men would have the opportunity of obtaining the highest scientific culture which was to be given. That he understood was the object and purpose and desire of the Master and Fellows of that College, in inviting persons like himself to take part in that great celebration; he presumed they wished them to understand that they recognised Science as a fundamental branch of human culture, and that they would do what in them lay to promote that happy commemoration to which he ventured to allude.

The toast was drunk with enthusiasm.

The Master briefly returned thanks, and stated that it had given them very sincere pleasure to entertain so illustrious an assembly, and expressed his deep regret that Sir George Paget was not with them.

The Rev. B. H. Drury proposed the health of the younger members of the College, saying that they were the life-blood of their College to-day, the source of their vitality, without whom they would have really little cause for existence.

Mr. Keeble, Natural Science Scholar of the College, made a short and graceful reply.

At the conclusion of dinner a move was made to the Combination Room, where friendly and animated intercourse was kept up for some time, and it was late before the last of those engaged in the celebration separated for the night.

Breakfast was provided the following morning from eight to ten for those resident in College overnight, and by midday the guests had departed, leaving the courts once more to solitude, and to their hosts a keen feeling of satisfaction at the honour done to the memory of William Harvey and to the College by the recent presence of so representative and distinguished a gathering of visitors.

SOME POINTS IN THE PHYSICS OF GOLF.[1]

III.

IN Part II of this paper (NATURE, Sept. 24, 1891) the following statements were made :—

"The only way . . . of reconciling the results of calculation with the observed data is to assume that, for some reason, the effects of gravity are at least partially counteracted. This, in still air, can only be a rotation due to undercutting."

"And, as a practical deduction from these principles, it would appear that, to secure the longest possible carry, the ball should be struck so as to take on considerable spin ———."

As these statements, and some of their consequences, have been strenuously denied, I must once more show at least the nature of the evidence for them.

It depends, in one of its most telling forms, upon the contrast between the length of time a well-driven ball remains in the air (as if in defiance of gravity) and the comparatively paltry distance traversed. Every one who thinks at all on the subject must see that, *without some species of support*, the ball could not pursue for six seconds and a half a course of a mere 180 yards, nowhere more than 100 feet above the ground.

In fact, if we assume the initial slope of the path to be 1 in 4, as determined for the average of fine drives by Mr. Hodge with his clinometer (NATURE, Aug. 28, 1890) the carry of a non-rotating ball will be approximately (in feet)

$$AgT^2,$$

where g is the acceleration due to gravity, T the time of flight in seconds, and A a numerical quantity depending on the resistance. The value of A varies continuously between the limits, 2 for no resistance, and 1 for infinitely great resistance. [It is assumed that the resistance is as the square of the speed.]

This formula gives, with the average observed value of T ($6^s\cdot5$, see Part II.) carries varying from about 900 down to 450 *yards!* The initial speed required varies from 416 foot-seconds upwards. The longest actually measured carry on record, when there was no wind, is only 250 yards. Unfortunately, in that case T was not observed, but analogy shows that it was probably much more than 7^s. Even if we take it as 7^s only, the "carry" ought to have been, by the formula (which is based on the absence of rotation), 522 yards at the very least!

I have purposely, in this example, kept to the case of an initial slope of 1 in 4; because those (and they are many, some of them excellent golfers) who altogether reject the notion that undercutting lengthens the carry, would of course in consistency refuse to believe that a long ball may sometimes start horizontally. But, to those who allow *this* statement, the fact that the action of gravity is occasionally largely interfered with, or even counteracted, is obvious without any numerical calculations. In fact, from my present point of view, initial slope is of little importance :—except, of course, in avoiding hazards. The want of it is easily made up for by a slightly increased rate of spin.

Another way of looking at the matter is to assume, from Mr. Hodge's data, 180 yards as a really fine carry, and thence to calculate by the formula the requisite time of flight. It varies from $4^s\cdot1$ to $2^s\cdot9$ according as the resistance, and therefore the necessary initial speed, are gradually increased; the former from *nil* to infinity, the latter from 132 foot-seconds upwards. Thus the observed time exceeds that which is really required when there is no spin, by 60 per cent. at the very least!

The necessity for underspin being thus demonstrated, we have next to consider how its effect is to be introduced in our equations. On this question I expressed a somewhat too despondent opinion in the previous part of this paper. A rather perilous mode of argument (which I have since been able to make much more conclusive) first suggested to me that the deflecting force, which is perpendicular at once to the line of flight and to the axis of rotation, must be at least approximately proportional to the speed and the angular velocity conjointly. But I tried (with some success) to verify this assumption by various experimental processes. These, as will be seen, led also to a numerical estimate of the magnitude of the deflecting force. [And I was greatly encouraged in this work by the opinion of Sir G. G. Stokes, who wrote :—"I think your suggestion of the law of resistance a reasonable one, and likely to be approximately true." This is quite as much as I could have hoped for.]

First: by the well-known phenomena called heeling, toeing, and slicing, which are due to the ball's rotation about a vertical axis. I have often seen a well-sliced ball, after steadily skewing to the right through a carry of 150 yards or even less, finally move at right angles to its initial direction, and retain very considerable spin when it reached the ground. Neglecting the effects of gravity, the equations of the path should be, in such a case,

$$\ddot{s} = -\frac{\dot{s}^2}{a},$$

$$\frac{\dot{s}^2}{\rho} = k\dot{s}\omega\,;$$

expressing the accelerations in the tangent, and along the radius of curvature, respectively. If we introduce the inclination, ϕ, of the tangent to a fixed line in the plane of the path, the second equation becomes

$$\phi = k\omega,$$

showing that the time-rate of change of direction is proportional to the speed of rotation. The first equation gives, of course,

$$\dot{s} = V\epsilon^{-\frac{s}{a}},$$

where V is the initial speed.

The space-rate of change of direction, *i.e.* the curvature of the path, is thus

$$\frac{d\phi}{ds} = \frac{k\omega}{V}\epsilon^{\frac{s}{a}},$$

increasing in the same proportion as that in which the speed of translation diminishes; and, if we regard ω as practically unaltered during the short time of flight, the intrinsic equation of the path is

$$\phi = \frac{ka\omega}{V}(\epsilon^{\frac{s}{a}} - 1).$$

A rough tracing from this equation is easily seen to reproduce distinctly all the characteristics of the motion

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[1] Part of the substance of a paper on the Path of a Rotating Spherical Projectile, read to the Royal Society of Edinburgh on June 5.